Mechanical Structures and Foundation Engineering 2010

International scientific conference MSFE 2010
Department of Mechanics of Materials,
Faculty of Mechanical Engineering,
VŠB - Technical University of Ostrava
September 13, 2010, Ostrava, Czech Republic

Nonlinear Analysis of Structure-Soil Interaction using Surface Subsoil Model

Nelineární Analýza Interakce Stavby s Podložím s Použitím Povrchového Modelu Podloží

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Abstract: Every building structure must be founded on a subsoil and sometimes this question is told: "is it possible to solve the structure separately and to replace an influence of subsoil by an estimation"? This contribution could be reply to the problem.

For some years we have been working on program SOILIN - Calculation of settlement and parameters of structure-soil interaction. This program can be used either individually to the prognosis of the settlements of an overloaded subsoil surface or as a special module of the FEM programs RFEM, or SCIA Engineer. In the last case program SOILIN calculates not only settlements but mostly the parameters C of the 2D subsoil model, too.

The used standards (ČSN, DIN EUROCODE) allow to specify the settlement only from the vertical normal stresses σ_z. Then the program SOILIN uses the Boussinesq solution of the infinite homogeneous and isotropic halfspace for a determination of state of stress in a subsoil and the course σ_z is extra modified with the excavation factor and the factor of underformable layer. Strains ε_z are solved generally by a nonlinear physical law $\varepsilon_z = f(\sigma_z)$. The function f can be very complicated, e.g. ČSN Standard introduces rigid-elastic stress-strain law using the so-called soil structure strength and DIN Standard assumes an experimentally obtained graphs ε_z versus σ_z. These physical subsoil models are included in the program SOILIN, too. In any case the solved problem is strong physical nonlinear. Therefore, no principle of linearity and superposition holds. Any level of loading, increasing or decreasing forms a new task. That is why the solution of structure-soil interaction is necessary to obtain with iteration. Firstly the FEM analysis of upper structure with initial C interaction parameters finds out contact stress. These values of contact stress are overloading for subsoil and perform as input for the SOILIN. This program solves settlements and corrects values of C parameters. Whole cycle FEM + SOILIN is repeated until the iteration test is fulfilled. In this way correct deformations and internal forces of construction are obtained.

On that account it is evident that structures on subsoil is necessary to solve such as the analysis of structure-soil interaction. Loads and own weight can be taken as acting directly on subsoil only then if the foundation and the interacting upper structure are extremely flexible. In any other case the subsoil is overloaded by the contact stress in the foundation-subsoil interface. That is the contact stress depends on stiffness of upper structure, loads, geometry of overloaded areas, geomechanical characteristics of subsoil, neighbouring, building, etc.

Abstrakt: Každá budova musí být založena na podloží a někdy může vyvstat otázka, zda je možné počítat konstrukci odděleně a nahradit podloží nějakým odhadem. Tento příspěvek by mohl být odpovědí na tento problém.

Řadu let jsme pracovali na programu SOILIN, který provádí výpočty sedání a parametrů interakce pro plošný model podloží. Tento program může být použit buď samostatně k prognóze sedání přitíženého povrchu podloží, nebo jako speciální modul programu RFEM nebo SCIA Engineer. Ve druhém případu program SOILIN nepočítá pouze sedání, ale počítá také parametry plošného modelu podloží.

Použité normy (ČSN, DIN EUROCODE) umožňují určit sedání pouze ze svislého napětí σ_z. Potom program SOILIN použije Boussinesqova řešení nekonečného homogenního a izotropního poloprostoru pro určení napětí v podloží. Takto získaný průběh napětí je modifikován výkopovým faktorem a faktorem

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nedeformovatelného podloží. Poměrné přetvoření ε_z je vypočítáno pomocí nelineárního konstitutivního vztahu $\varepsilon_z = f(\sigma_z)$. Funkce f může být velmi složitá, např. norma ČSN uvádí ideální tuhoelastický diagram s použitím tzv. strukturní pevnosti a norma DIN předpokládá experimentálně získané vztahy mezi σ_z a ε_z. Tyto konstitutivní modely podloží jsou zabudovány v programu SOILIN. V každém případě je řešení silně fyzikálně nelineární, tudíž zde neplatí princip superpozice. Jakékoliv zvětšení, nebo zmenšení napětí znamená novou úlohu. Proto je nutné řešení problému interakce stavby s podložím provádět iterativně. Nejdříve se provede řešení horní stavby při počátečním nastavení parametrů podloží C, které určí kontaktní napětí. Takto získané kontaktní napětí slouží jako přitížení povrchu podloží pro program SOILIN, který vypočítá sedání a opravené hodnoty parametrů C. Celý cyklus FEM + SOILIN je opakován až do splnění testu na ukončení iterací. Tímto způsobem jsou korektně získány hodnoty deformace a napětí v konstrukci.

Na popsaném postupu je zřejmé, že konstrukce na podloží je nutno řešit jako interakci stavby s podložím. Zatížení a vlastní váha budovy může být použito jako zatížení povrchu podloží jen v případě extrémně poddajné horní konstrukce. V jakémkoliv jiném případě musí být podloží zatíženo kontaktním napětím na rozhraní stavby s podložím, které závisí na tuhosti horní stavby i podloží, zatížení, geometrii kontaktu stavby s podložím, geomechanických parametrech podloží, zatížením sousedními budovami, atd.

Keywords: Structure-Soil interaction, finite element method, analysis of structures

Klíčová slova: Interakce stavby s podložím, metoda konečných prvků, analýza konstrukcí.

1. Introduction

As it follows from the previous explanation, certain assumptions must have been introduced in the reduction of the 3D subsoil massif to the surface model: for example, the damping function f that determines the distribution of the settlement of a point on the subsoil surface over the depth. It can be easily shown that this distribution will differ significantly for different points on the foundation base and it will depend on the size and distribution of loads, even if the material of the subsoil is linear. However, this is not generaly the case in the subsoil under structures. The parameters of the surface models are a function of (i) the position of the point on the subsoil surface, (ii) properties of subsoil material, and also (iii) the stiffness of the superstructure and (iv) loads.

Therefore, the design and assessment of any structure that is in the contact with subsoil must deal with the interaction of the structure, foundation and subsoil. The load the foundation is subjected to is not transferred to the subsoil surface directly (if so, it would be possible to impose the corresponding reaction back to the foundation), but it depends on the distribution of the contact stress across the foundation base. This distribution, however, does not depend just on the load, but also on the relative stiffness of the foundation and superstructure in relation to the subsoil, on physical properties of the subsoil (heterogeneity, geological fractures), on adjacent constructions, etc.

This chapter describes the procedure that makes it possible to apply the surface model of the structure-soil interaction and, at the same time, to take into account the above-mentioned sources of nonlinearity. This procedure enables to take into consideration the several current codes of practice that define (i) the material properties of subsoil and (ii) the approach for the calculation of settlement of structures.

2. Stress in subsoil

The initial problem in determining the “support conditions” of foundation structures is to find out the stress-state in the subsoil that, in general, covers the whole soil massif under the structure and in the vicinity that can influence the behaviour of the building on the foundation in question. The widespread approach (which is also implemented in numerous currently valid standards) today is that the stress-state in the subsoil can be obtained using the model of a Boussinesq ideal homogenous half space. The previously used Winkler model did not make it possible to express (i) the decrease in the stress that occurs with the increasing depth and (ii) the formation of the subsidence basin, not mentioning (iii) the mutual impact of individual structures on each other. The application of the stress-state analysis using the half space inevitably raised questions concerning the conditions under which the solution is still satisfactory and under which a more detailed approach must be applied. In practice, the subsoil is almost always vertically non-homogenous (stratified). Quite often, the engineer has at hand geotechnical data that clearly prove also the horizontal heterogeneity. Therefore, various surveys have been performed to document tens of percents in the difference in stress σ_z in case that soft and rigid strata alternate in the subsoil. Already in 1990 similar comparison [1] were carried out in which the impact of non-homogeneity of the half space on the stress tensor field due to varying thickness of inserted strata with two-, five- and ten-times greater deformation modulus E in comparison with the remaining strata were analysed. A set of calculations was performed for a prismatic problem, which revealed that if, for example, the top stratum of the geological profile features larger E, the damping of the axial stress component σ_z is faster and the difference between these values and the results for homogenous half space reaches up to 30%. On the other hand, if the stratum with the greater E is inserted in between other strata, the damping of σ_z in higher-located strata is slower (the zone in question is "clamped" in between the loaded foundation base and the rigid stratum). It means that in the case of stratified geological profile with larger difference between the deformation modulus E in individual strata, it is more economical, and sometimes even safer, to analyse the stress-state of this non-homogenous half space. Naturally, it is clear that no such analyses will be performed in common practice. Therefore, the authors of technical standards included into the standards the article that the model of the ideal homogenous half space can be used also for non-homogenous and anisotropic subsoil (see e.g. art. 72, ČSN 73 1001 [2]. With regard to the fact that the error in the input geomechanical values required for the calculation of settlement is significantly greater than the error due to less accurate calculation of stress, the half space approach can be used for the calculation of σ_z without scruples (being aware of possible deviations in the case of strata with significantly different deformation modulus E).

3. Physical model of soil based on the formula stated in CSN 73 1001

The calculation model of subsoil is defined in art 116 – 122 of the above-mentioned standard [2]. There is no sense in rewriting the related paragraphs. We rather should try and comment on the provisions within the context of their application for the interaction of buildings with soil environment.

The standard stipulates in page 33 the formula for the calculation of settlement s of the subsoil surface:

$$s = \sum_{i=1}^{n} \frac{\sigma_{z,i} - m_i \sigma_{or,i}}{E_{oed,i}} h_i \tag{1}$$

where $\sigma_{z,i}$ is the vertical axial component of stress in an elastic homogenous infinite half space (or stratum), $\sigma_{or,i}$ is the analogous component of the original geostatic stress, m_i is the correction coefficient for surcharge loading (coefficient of structural strength), $\sigma_{s,i}$ is the structural strength (i.e. $m_i \sigma_{or,i}$) and n is the number of strata with thicknesses h_i and constrained (oedometric) modulus $E_{oed,i}$ in which the *effective stress* is non-negative:

$$\sigma_{zú} = \sigma_{z,i} - \sigma_{s,i} = \sigma_{z,i} - m_i \sigma_{or,i} \geq 0 \qquad (2)$$

Zero deformation is assigned to regions where the effective stress is negative. This represents mainly larger depths where the subsoil does not deform any more. The condition of zero effective stress then determines what is termed the depth of the deformed subsoil zone. The situation is, however, a bit more complex in regions outside of the foundation base. Even though the magnitude of the vertical axial component σ_z is zero on the surface, it increases in deeper strata due to the effect of shear distribution - see fig.1. The condition of zero effective stress thus determines the stratum of the deformed subsoil zone in which the upper level of the positive effect of the stress is not on the subsoil surface.

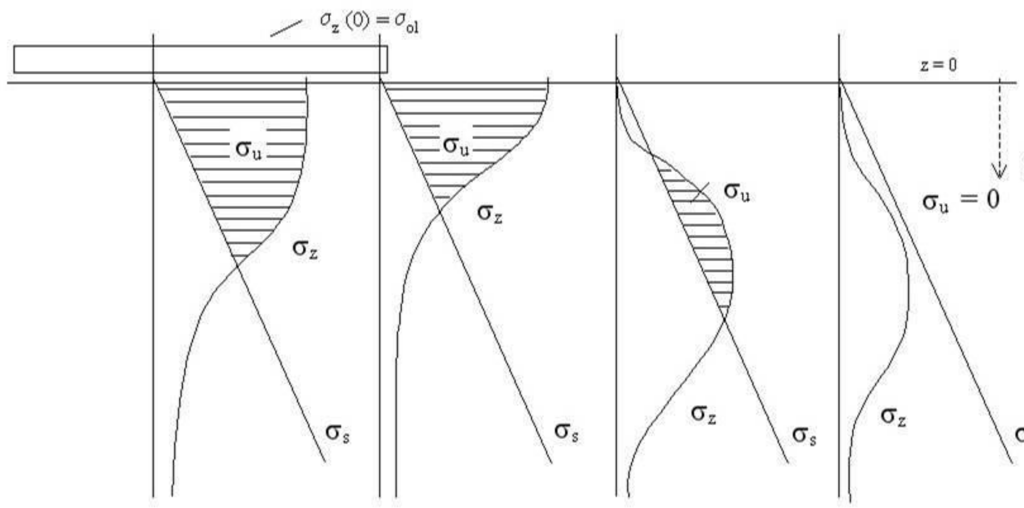


Fig.1 Effective stress

The basic formula (1) for the calculation of settlement was derived in such a way that the standard replaced the integral over the vertical axis z

$$s = \int_0^H \varepsilon_z(z) dz \qquad (3)$$

by summation using the "rectangular rule" with the division of the interval $0 \leq z \leq H$ into n parts that may be generally of different size. The values of $\sigma_z, \sigma_{or}, m, E_{oed}$ with the index i relate to the middle horizons of the strata. The accuracy of the summation can be increased through finer mesh, i.e. greater n.

Stress σ_z is calculated from surcharge loading σ_{ol} in the foundation base that must be in practice analysed mainly by means of iteration, i.e. through the solution of a contact problem: building + foundation + subsoil. The load of the foundation would be transferred directly to the subsoil surface only in case of a very flexible foundation. The result functions representing the distribution of the contact stress across the foundation base are, in fact, generally statically indeterminate quantities. As already stated, they depend not only on the load, but also on the relative stiffness of the foundation with the superstructure in relation to the subsoil, on physical properties of the subsoil, on the time factor (changes during construction and service life, consolidation), etc. The foundation base is subjected to the load directly only in special situations (e.g. load from embankments) and if this happens, the surcharge loading is known in advance.

Stress σ_{or} is determined by the weight of the soil above the given point that is in the middle of the thickness of the stratum. The calculation requires that the values of the effective unit weight (i.e. above the groundwater level) are known and it is assumed that its distribution is constant in every stratum.

Considering formulas (1) and (2), it is clear that our standard defines for each stratum a rigid-elastic stress-strain diagram with singularity in point $\sigma_z = \sigma_s$ where the rigid model becomes elastic – see fig.2.

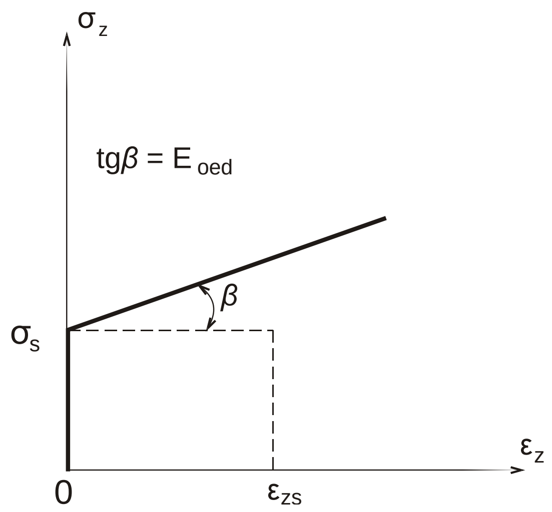


Fig.2 Stress-strain diagram of soil according to CSN

This strong physical nonlinearity results in the fact that the principles of linearity and superposition do not apply to the calculation of settlement and that it is not possible to use the same approach for the definition and assessment of the structure as in linear analysis. It would not be possible to evaluate separately individual load cases or use an automatic selection of the most effective combinations for each result quantity. Fortunately enough, the problem of the structure-soil interaction has certain specifics that allow for weakening of this strict limitation. Short-term variable loads involved in the load combinations do not produce (through their effects) settlement of the subsoil and, therefore, they do not have to be included into the complete loading for which the support parameters will be evaluated. The principle of the solution is that the long-term loads are concentrated into one of just a few load cases and the supporting conditions are calculated for this linear combination. During the evaluation we have to bear in mind that the results correspond only to the given total load and not to individual load cases. Concerning the short-term loads, these are included only into the formulas for the analysis of the most effective combinations. They may have an impact on the deformations and internal forces of the superstructure, but not on the calculation of the interaction parameters.

However, this approach is not recommended for problems in which the stress in the foundation base is influenced mainly by variable loads (e.g. light-weight steel halls) and the C^S parameters must be calculated for each nonlinear combination separately.

4. Physical model of soil according to DIN 4019

Settlement is in the German standard [3] calculated in the following way

$$s = \sum_{j=1}^{n} ds_j, \qquad ds_j = \varepsilon_{z,efj} dz_j, \qquad \varepsilon_{z,efj} = \frac{\sigma_{z,efj}}{E_{sj}} \tag{4}$$

where:

ds_j compression of the j-th layer of the subsoil from the given surcharge loading

j number of the layer with thickness dz_j

n total number of layers into which the deformed zone of the subsoil from the loaded place up to the limit height H_m is divided; geological strata represent a coarse division that is refined for mathematical reasons (numerical integration) similarly to formula (1) for CSN

$\varepsilon_{z,efj}$ relative compression of the j-th layer from the given surcharge loading, it is the vertical component of deformation εz called in DIN 4019 "*specific settlement*" and marked s'_z, which reminds derivational definition of $\varepsilon_z = ds/dz$.

$\sigma_{z,efj}$ vertical component of stress σ_z in the centroidal level of the j-th layer due to the given surcharge loading.

ef index used to emphasise that we deal with the effect of the given surcharge loading, because DIN 4019 introduces stress-strain diagrams $\sigma_z - \varepsilon_z$ valid for the complete stress- and deformation-state of the stratum including the initial geostatic stress-state and corresponding deformation

E_{sj} this is termed "average modulus of deformation" of the j-th layer that corresponds to the current stress-state and deformation of that layer. DIN 4019 defines it as a secant modulus of the graph $\sigma_z - \varepsilon_z$ between two points: the original geostatic stress-state $(\sigma_{z,or}, \varepsilon_{z,or})$ and the stress-state after applying the surcharge loading $(\sigma_{z,or}, \sigma_{z,ef}, \varepsilon_{z,or} + \varepsilon_{z,ef})$ - see fig.3.

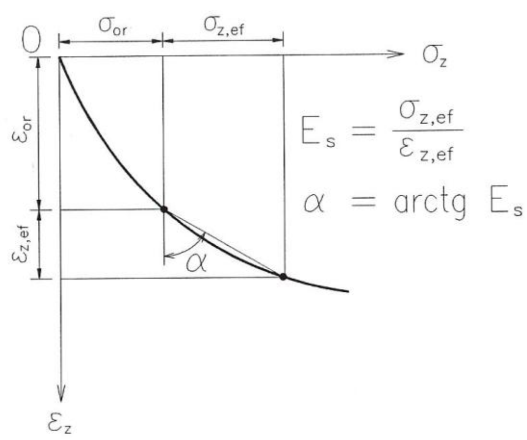


Fig.3 DIN – secant modulus E_s

Additionaly, following issues might be mentioned:

a) The standard assumes that the stress-strain diagram $\sigma_z - \varepsilon_z$ (fig.3) is provided for the calculation of settlement for every geological stratum starting from the initial state $\sigma = 0, \varepsilon_z = 0$ and ending with the state σ_z that is at least equal or greater than the expected state $(\sigma_{z,or} + \varepsilon_{z,or})$. It is apparently a uniaxial constrained deformation as in CSN.

b) According to DIN 4019, the j-th layer first gets into the state of the original geostatic stress-state (which corresponds to $\sigma_{z,or}$ from CSN standard) and this happens along the same path as given in the diagram (fig.3), i.e. by monotonous increase of σ_z. This defines the first point of the graph $(\sigma_{z,or}, \varepsilon_{z,or})$. Then, the surface is subjected to the surcharge loading due to the structure and both stress and deformation increase. The second point of the graph $(\sigma_{z,or}, \sigma_{z,ef}, \varepsilon_{z,or} + \varepsilon_{z,ef})$ is reached. What can be observed on the surface, i.e. settlement s, is just the effect of surcharge loading $(\sigma_{z,ef}, \varepsilon_{z,ef})$. That means that we are interested only in the secant modulus E_s defined by the line connecting the two mentioned points of the graph. DIN draws stress σ_z along the horizontal axis and deformation ε_z ("specific variable settlement s'_z") along the vertical axis downwards.

c) It is recommended to perform the summation (4) only within the limit depth H_m defined by the requirement

$$\sigma_{z,ef} = 0.2\sigma_{z,or} \qquad \left(german \ i\sigma_1 = 0.2\sigma_{\ddot{u}}\right) \tag{5}$$

which corresponds to the assumption that in depth H_m the coefficient m always equals 0.2 ($m = 0.2$ - see the coefficient of structural strength from CSN).

d) Stress σ_z due to the surcharge loading of the surface σ_1 is in DIN 4019 mentioned only in the form of the following formula

$$\sigma_z = i\sigma_1 \tag{6}$$

where "i" is the coefficient found in some table of results for a sequence. About 30 collections of tables by different authors are recommended and they limit only to a rectangular or circular surcharge loading area. A warning is given concerning various difficulties related to table-defined parameters. Therefore, it is more convenient to use directly the exact solution of the stress tensor for the elastic homogenous half space, as implemented in the CSN standard.

e) If the foundation is made in an excavation, the surcharge loading of the foundation base is reduced by the total original weight of the excavation $\sigma_{z,or,v}$ and, consequently, only the following load is taken into account

$$p = \sigma_{ol} - \sigma_{z,or,v} \qquad (german \ \ \sigma_1 = \sigma_o - \gamma d) \tag{7}$$

This would correspond to CSN 73 1001 in a fictive situation with $m = 1$ for a stratum located at the foundation base in depth d. It means, that, most likely, only shallow excavations are considered.

f) Graphs required in points a), b) assume the possibility to perform an experiment with a specimen of the soil from each stratum. At the beginning of the experiment the specimen must be in the ideal initial state of released stress and deformation and the subsequent process must follow exactly the same path as in reality (in situ). Such an experiment is impossible in terms of time, even though the geostatic stress-state itself could arise trough monotonous increase of load, e.g. in sediments. It is clear that the graphs required in DIN 4019 must include professional experience and possibly adaptation for a given locality with its geological history taken into account. Therefore, DIN 4019 requires that the graph be produced by an established soil-mechanics laboratory.

Brief summary: The well known uncertainty following from the nature of geomechanical problems is in CSN 73 1001 implemented since 1988 mainly through the coefficient of structural strength $m\left(0.1 \le m \le 0.5\right)$ and modulus E_{oed}. In DIN 4019, this uncertainty is included into the graphs (stress-strain diagrams of soils as virgin materials).

However, the German design practice often avoids using these graphs directly in favour of their cumulative consequences, for example in the form of "*average modulus* E_s *in the analysed problem*".

5. Physical model of soil according to Eurocode 7

The EC 7 [4] standard is rather tolerant and does not prescribe particular types of settlement calculation. It only recommends performing the calculation of stress using a homogenous half space and considering the value of the limit depth as defined by formula (5), the same way as stated in the DIN standard.

As the recommended magnitude of the limit depth is defined by the value equal to 0.2 of the initial geostatic stress-state, we suggest that the calculation of the compression of soil strata according to EC be performed using the CSN variant with the coefficient of structural strength equal to the same value, i.e. $m = 0.2$. The introduction of the limit depth means that the deformation modulus equal to infinity is assigned to deeper strata, which means zero compression. The strata just above should thus feature the minimum compression (in order to prevent the introduction of an unjustified singularity of the distribution of compression into the model), which is best suited just by the introduction of the term "structural strength". This makes the formula for the determination of the limit depth logical and justified.

Concerning the excavations, we tend to support the idea that it is not suitable to deduct the weight of the excavation in determining the surcharge loading, i.e. we prefer to keep the value of the surcharge loading the same as if it acted on the original terrain.

6. Reduction of the dimension of the interactive problem

In civil engineering practice, calculation models of structures are mostly created from planar and beam (finite) elements, i.e. 2D and 1D. The subsoil as soil environment is, however, a typical 3D medium and it should be analysed that way (i.e. 3D). In general, the system (structure + foundation + subsoil) is 3D in nature and if we wanted to know in detail the stress-state and deformation below the foundation, we would have to model the subsoil using 3D finite elements. This would, on the other hand, disproportionally increase the number of unknown parameters of deformation and – in practical models – we would exceed the time and capacity limits of contemporary computers. Moreover, if the application of 3D finite elements was driven by the attempt to perform a more detailed analysis, such a solution would make disproportionally big demands on the physical input data and, as a result, the geological survey would strongly increase the total costs of the whole project. Fortunately enough, the primary goal is the design of the structure and foundations and we are interested in the conditions in the subsoil only to be able to determine its effect on the response of the structure. In that situation we can swap to a solution in which the whole 3D subsoil is represented just by its 2D surface.

7. Surface model of subsoil

The least credible is the Winkler model that considers the subsoil to be an infinitely dense system of springs or thick liquid. This model is not capable of expressing the creation of a subsidence basin or co-action of neighbouring buildings. The Winkler model was improved by Pasternak who (in order to give a true picture of shear components that emerge in non-uniform settlement) established the second constant C_2^S.

At around 1975, V. Kolar and I. Nemec introduced a new concept for this issue – see [5]. This is based on creation of such a 2D surface model the deformation of which produces the same virtual work as in 3D subsoil. With this, a whole hierarchy of parameters can be built (fig.4).

The most important are:

- C_1^S parameters of the interaction of the foundation with the surface 2D model of the subsoil in physical relations containing components of displacement u, v, w.

Winkler formula for vertical components:

$$\sigma_z = r\,[\text{kPa}] \quad = C_{1z}^S\,[\text{MNm}^{-3}] \cdot w\,[\text{mm}] \tag{8}$$

for horizontal shear components:

$$\tau_{zx} = s_x\,[\text{kPa}] \quad = C_{1x}^S\,[\text{MNm}^{-3}] \cdot u\,[\text{mm}] \tag{9}$$

$$\tau_{zy} = s_y\,[\text{kPa}] \quad = C_{1y}^S\,[\text{MNm}^{-3}] \cdot v\,[\text{mm}] \tag{10}$$

- C_2^S parameters of the interaction of the foundation with the surface 2D model of the subsoil in physical relations containing the first derivative of settlement.

Pasternak formula for shear forces:

$$t_x\,[\text{kNm}^{-1}] = C_{2x}^S\,[\text{MNm}^{-1}] \cdot \partial w/\partial x\,[\text{mm/m}] \tag{11}$$

$$t_y\,[\text{kNm}^{-1}] = C_{2y}^S\,[\text{MNm}^{-1}] \cdot \partial w/\partial y\,[\text{mm/m}] \tag{12}$$

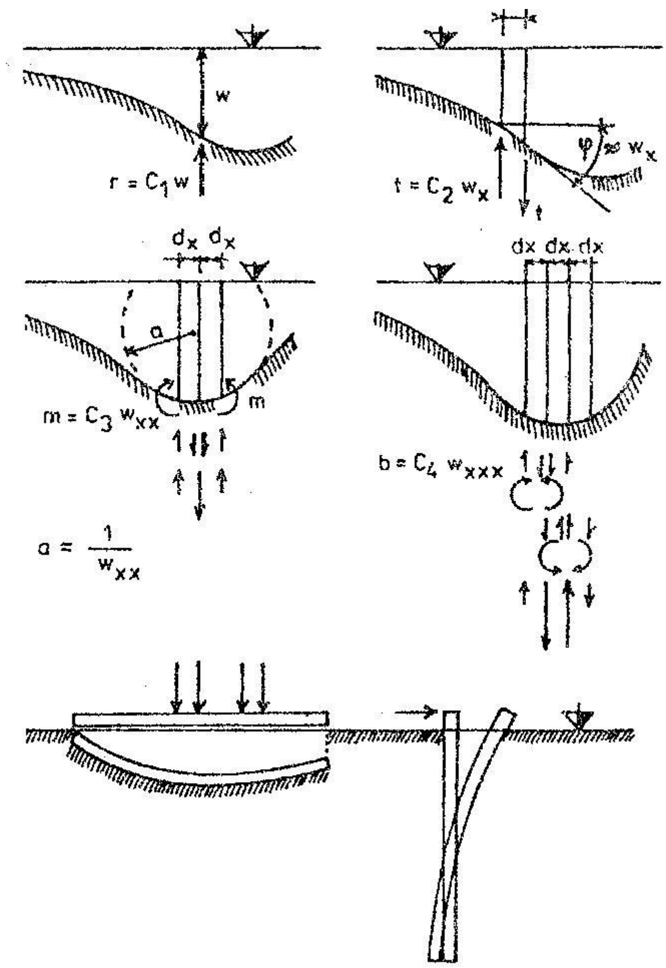


Fig.4 Physical meaning of some C parameters

This new model of the subsoil is described in detail in [5] and, therefore, in this text we limit ourselves to a brief derivation for the purpose of the explanation that will follow:

The formula for the potential energy of internal forces of the 3D model has the following form:

$$\Pi_{3D}^{i} = \frac{1}{2}\int_{V} \underline{\boldsymbol{\sigma}}^{T}\, \underline{\boldsymbol{\varepsilon}}\, dV = \frac{1}{2}\int_{V} \underline{\boldsymbol{\varepsilon}}^{T}\, \underline{\mathbf{D}}\underline{\boldsymbol{\varepsilon}}\, dV \tag{13}$$

Neglecting the effect of horizontal components of deformation, we get the following vectors:

$$\boldsymbol{\sigma} = \left[\sigma_z, \tau_{zx}, \tau_{yz}\right]^T = \mathbf{D}\boldsymbol{\varepsilon} \tag{14}$$

$$\boldsymbol{\varepsilon} = \left[\varepsilon_z, \gamma_{zx}, \gamma_{yz}\right]^T = \left[\frac{\partial w}{\partial z}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right]^T \tag{15}$$

This means the corresponding simplification of the matrix of physical constants $\mathbf{D}$.

$$\mathbf{D} = \begin{bmatrix} E_z & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & G \end{bmatrix} \tag{16}$$

In order to be able to reduce the problem from 3D to 2D, it is necessary to integrate formula (13) over the z-axis. For this reason, a certain “damping function” $f(z)$ is introduced and it is defined by the ratio of the settlement in the given depth to the settlement of the surface $w_0(x,y)$

$$f(z)=\frac{w(x,y,z)}{w_0(x,y)} \tag{17}$$

Modifying (15) and (17) we get

$$\boldsymbol{\varepsilon}=\left[w_0(x,y)\frac{\partial f(z)}{\partial z},\frac{\partial w_0(x,y)}{\partial x}f(z),\frac{\partial w_0(x,y)}{\partial y}f(z)\right]^T \tag{18}$$

Substituting (18) into the formula for the potential energy of body $V=\Omega H$, where Ω is the extent of the 2D model and H is the depth of the deformed zone of the 3D model, we obtain the following formula:

$$\begin{aligned}\Pi^i_{2\mathrm{D}}&=\Pi^i_{3\mathrm{D}}=\frac{1}{2}\int_V\left[\sigma_z\varepsilon_z+\tau_{zx}\gamma_{zx}+\tau_{yz}\gamma_{yz}\right]dV=\\&=\frac{1}{2}\int_V\left[\varepsilon_z^2E_z+(\gamma_{zx}^2+\gamma_{yz}^2)G\right]dV=\\&=\frac{1}{2}\iint_\Omega\left[w_0^2\int_0^H E_z\left(\frac{\partial f}{\partial z}\right)^2dz+\left(\frac{\partial w_0}{\partial x}\right)^2\int_0^H f^2Gdz+\left(\frac{\partial w_0}{\partial y}\right)^2\int_0^H f^2Gdz\right]d\Omega\end{aligned} \tag{19}$$

Integrating over z, we get the formula for the potential energy of internal forces of the 2D model with two parameters: C_1^S, C_2^S

$$\Pi^i_{2\mathrm{D}}=\frac{1}{2}\iiint_\Omega\left[C_{1z}^Sw_0^2(x,y)+C_{2x}^S\left(\frac{\partial w_0(x,y)}{\partial x}\right)^2+C_{2y}^S\left(\frac{\partial w_0(x,y)}{\partial y}\right)^2\right]d\Omega \tag{20}$$

Comparing (19) and (20), we can define the relation between the parameters of the general (3D) and surface (2D) model:

$$C_{1z}^S=\int_0^H E_z\left(\frac{\partial f(z)}{\partial z}\right)^2dz \qquad C_{2x}^S=C_{2y}^S=\int_0^H Gf^2(z)dz \tag{21}$$

In this interpretation, the surface model has been implemented into the RFEM system [6] in such a way that the energy accumulated in the subsoil is added to the potential energy of the structure. What remains to be answered is how the appropriate C^S parameters can be obtained with the best possible accuracy. This can be achieved using the SOILIN module (see [6]) that – on the basis of the stress-state of the elastic homogenous half space and standard-defined model – determines in any location the distribution of settlement and subsequently the sought-after C^S

parameters. The calculation of parameter C_{1z}^S is not carried out according to formula (21), but using directly the values of stress σ_z and strain ε_z from the stress-strain diagram, i.e. through the comparison of area density of the energy corresponding to compression ε_z for 3D model and 2D model, which represents the first members in the expressions for the potential energy in formulas (19) and (20). That means

$$\frac{1}{2}\int_0^H \sigma_z \varepsilon_z dz = \frac{1}{2} C_{1z}^S w_0^2(x,y) \tag{22}$$

Modifying this we get the following formula for the calculation of parameter C_{1z}^S

$$C_{1z}^2 = \frac{\int_0^H \sigma_z \varepsilon_z dz}{w_0^2(x,y)} \tag{23}$$

As the similar procedure (i.e. the determination of slopes γ_{zx} and γ_{yz}) applied to the calculation of parameters C_2^S would cause numerical complications due to the necessity to determine other settlement values in ambiguously obtainable differences, it was decided to perform the calculation of parameters C_2^S by means of what is termed "isotropic form" using formulas (17) and (21).

$$C_{2x}^S = C_{2y}^S = \int_0^H G f^2(z) dz = \frac{\int_0^H G w^2(x,y,z) dz}{w_0^2(x,y)} \tag{24}$$

As the C^S parameters influence the contact stress and, at the same time, the distribution of the contact stress has an impact on the settlement of the foundation base and thus also on the C^S parameters, it is necessary to perform the calculation of the structure-soil interaction in an iterative way – see fig.5.

Therefore, the calculation of the superstructure and the determination of the C^S parameters are performed in turns and, in majority of realistic problems, the relative concord can be obtained from an arbitrary initial assumption. Suitability of the initial assumption influences only the number of iteration steps. The results of the calculation are internal forces and deformations of the structure, settlement of the subsoil surface, contact stress in the foundation base in individual iteration steps and the final interaction parameters C^S .

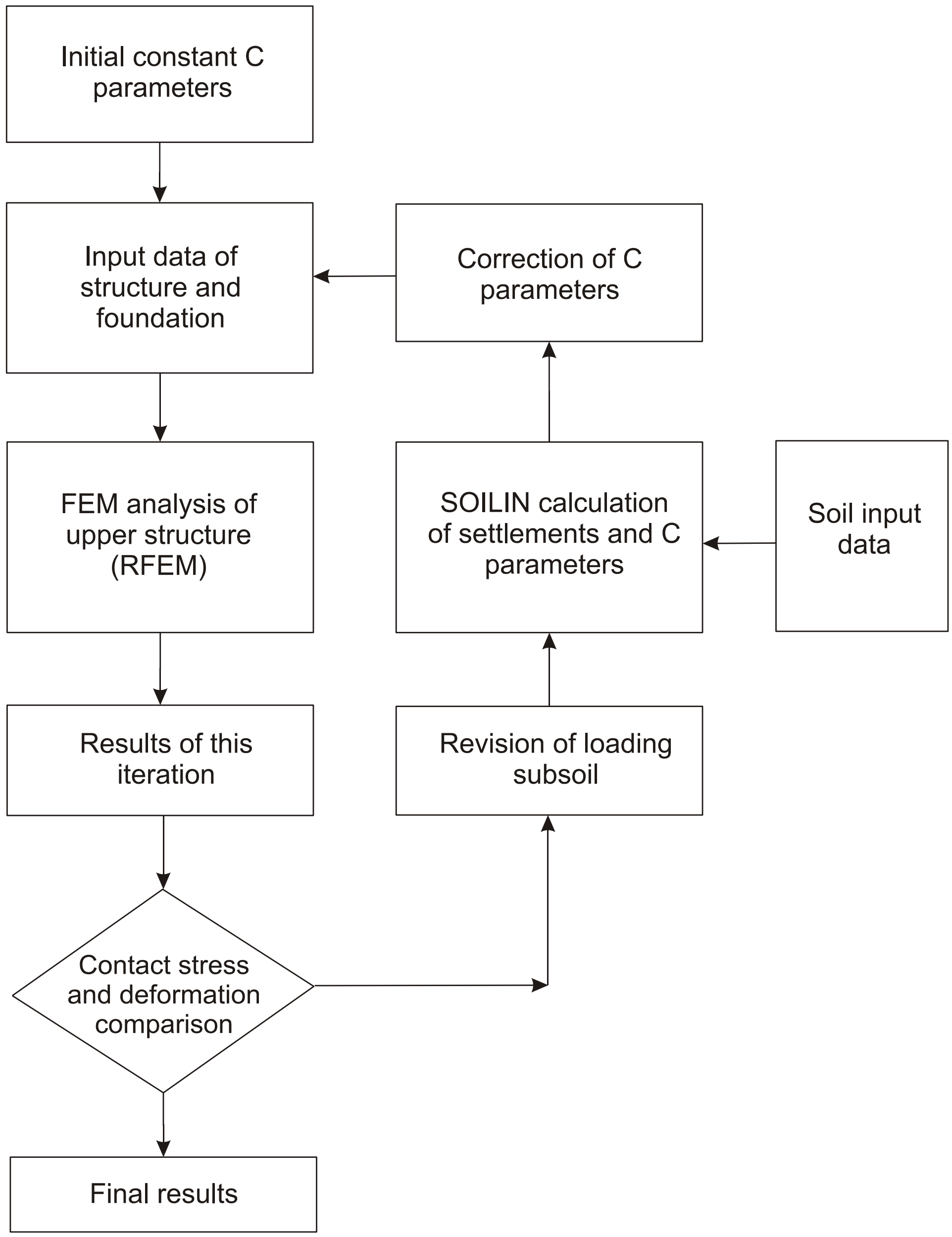


Fig.5 Scheme of the iterative calculation

8. The effect of subsoil outside of the structure

The subsidence basin of the subsoil does not end at the boundary of the foundation structure, but it extends further outside of the foundation. The reason is that the transfer of shear through components τ_{zx} and τ_{yz} produces vertical axial stress σ_z even in the subsoil outside of the foundation base. It means that deeper subsoil strata are compressed in the location that is not subjected to any surface load. As a result, it is the value of the structural strength σ_s that determines the boundary of the subsidence basin.

This phenomenon is coped with in the SOILIN program through the approach in which the stress-state in the subsoil is solved according to the standard-defined half space model exactly using the analytical formulas in an arbitrary point (x, y, z), i.e. even outside of the foundation base. The appropriate geological profile is taken into account in the given location and, therefore, the settlement of the surface and the interaction parameters C^S can be determined. And just the C_2^S parameters that appear in the physical formulas containing the first derivative of settlement (it is in fact the elastic resistance of the surrounding against skewing) allow in the model of the superstructure for the transfer of shear in the subsoil, which is successfully exploited in practice.

The subsoil outside of the foundation itself can be taken into account, in general, in two ways:

1. Planar “subsoil” macroelements of an insignificant thickness are defined around the slabs modelling the foundation. The iteration process will calculate the settlement and the corresponding C^S parameters even in these locations, which ensures correct supporting conditions of the edges of the foundation structure. In addition, this approach makes it possible to obtain an idea about the size of the subsidence basin.

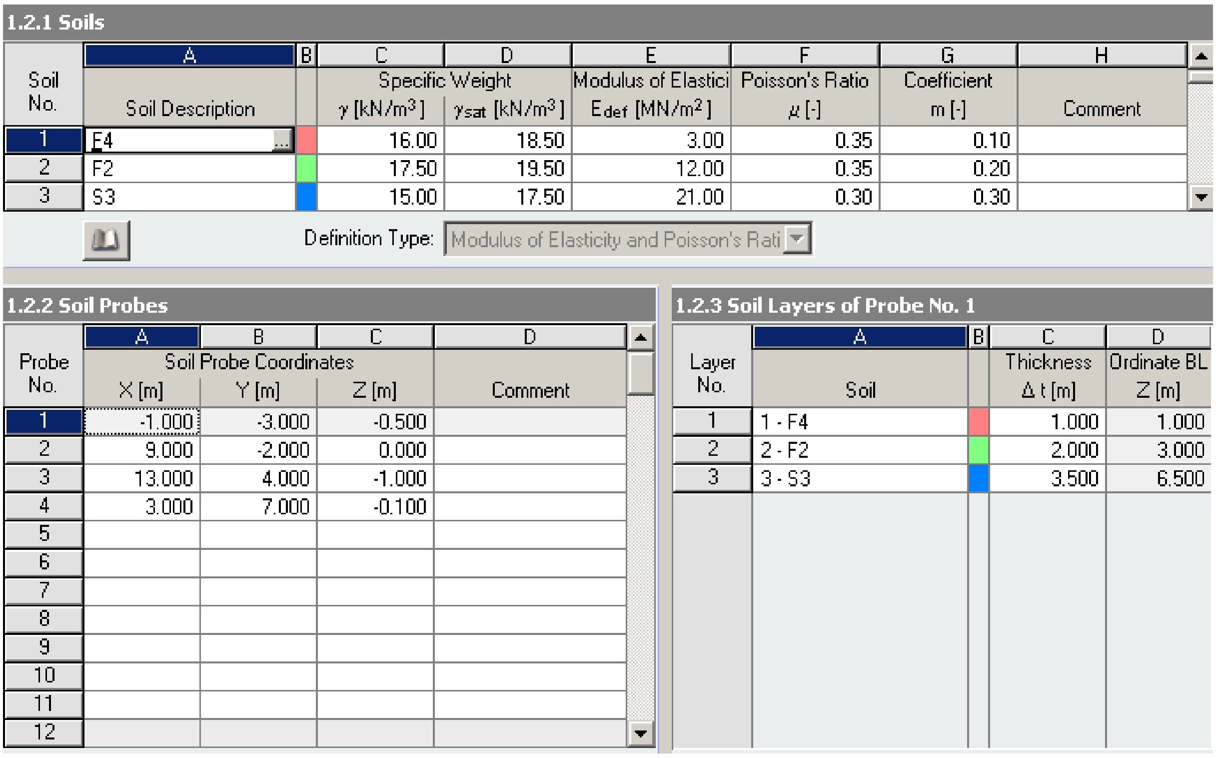
2. Newer versions of the SOILIN module make the necessary steps to consider the effect of the surrounding subsoil automatically. The program detects the edge of a continuous region (or several regions) on the subsoil and vertical supports are assigned to the nodes at these edges. In every iteration, the stiffness of these springs is calculated from the just determined C^S parameters of those elements that adjoin the given node. It is an approximate modelling, but the results are close to the results of variant 1. Moreover, the calculation is faster as it is not necessary to add “subsoil” elements. If the effect of the surrounding subsoil is not to be considered at a specific edge (e.g. in the vicinity of a sheet pile wall), it can be achieved through the input of a spring with a small stiffness. Such an input at a corresponding line overwrites the springs generated in SOILIN.

The consideration of the effect of the subsoil outside of the foundation structure can have a significant impact on the behaviour of the whole structure. Simply said, the absolute value of the accompanying settlement decreases, but the relative settlement and internal forces increase, which is caused by the more definite support conditions at the edges of the foundation structure. The modelling of the interaction is more accurate, however, often at the cost of higher number of iterations. It can also happen that, due to singularities occurring at the boundary of the foundation, the required convergence does not happen even for increased maximum number of iteration. Then it is necessary to closely scrutinise the results of e.g. two consecutive iterations and make a “professional estimate”.

9. Implementation into RFEM system

First of all, a linear combination of long-term load cases or a nonlinear combination must be defined and this combination must be selected in the settings for the calculation of the structure-soil interaction.

The geological profile is defined by means of geological bores directly in the location of drill holes. For that purpose, the geological profiles must be defined and several geological strata (see fig.6) can be assigned to them. These are characterised by the stratum thickness, modulus of deformation E_{def}, Poisson coefficient of transverse contraction ν, unit weight of dry and wet (saturated) soil γ and coefficient of structural strength m (for the CSN standard). If the water table is found in a certain depth, this distance from the top of the bore can be specified and the unit weight of wet soil reduced by 10 kN/m^3 is automatically assigned to all the strata located below the water level. If the option “Non-compressible soil” is selected, the program automatically introduces the depth reduction coefficient according to CSN. Numerically it means that the damping of stress component σ_z of the elastic half space is slowed down.



1.2.1 Soils

Soil No.	A Soil Description	B	C Specific Weight γ [kN/m^3]	D γ_{sat} [kN/m^3]	E Modulus of Elastici E_{def} [MN/m^2]	F Poisson's Ratio μ [-]	G Coefficient m [-]	H Comment
1	F4		16.00	18.50	3.00	0.35	0.10	
2	F2		17.50	19.50	12.00	0.35	0.20	
3	S3		15.00	17.50	21.00	0.30	0.30	

Definition Type: Modulus of Elasticity and Poisson's Rati

1.2.2 Soil Probes

Probe No.	A Soil Probe Coordinates X [m]	B Y [m]	C Z [m]	D Comment
1	-1.000	-3.000	-0.500	
2	9.000	-2.000	0.000	
3	13.000	4.000	-1.000	
4	3.000	7.000	-0.100	
5				
6				
7				
8				
9				
10				
11				
12				

1.2.3 Soil Layers of Probe No. 1

Layer No.	A Soil	B	C Thickness Δt [m]	D Ordinate BL Z [m]
1	1 - F4		1.000	1.000
2	2 - F2		2.000	3.000
3	3 - S3		3.500	6.500

Fig.6 Geological profile

The corresponding geological profile is positioned through the x, y, z global coordinates that denote the point on the surface of the terrain (see fig.7).

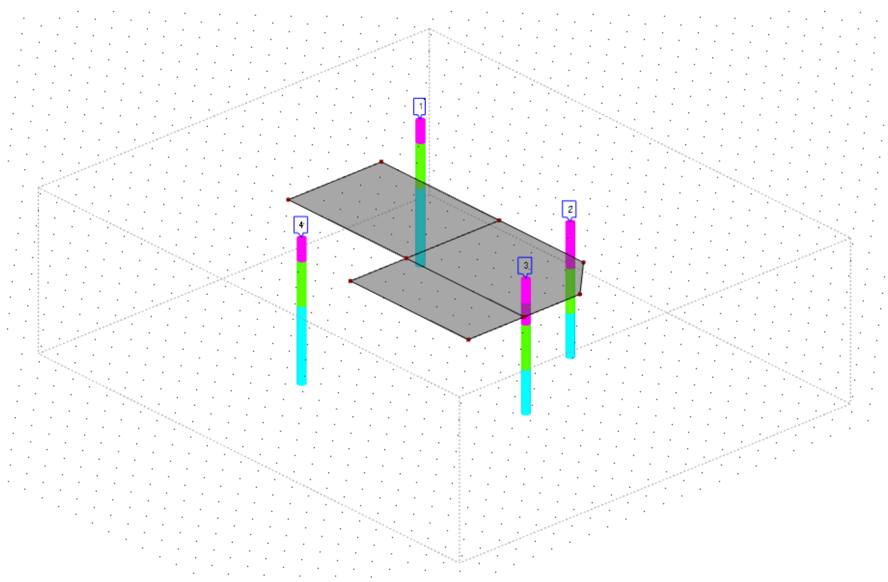


Fig.7 Location of bores

Therefore, the position of terrain must be harmonised with the already input superstructure, so that the global z-coordinates match each other. It is possible to define an arbitrary number of bores (bore-hole profiles) that must, however, contain the same number of geological strata. If some of the geological stratum is missing in a certain geological profile (it diminished), it still must be included in the corresponding geological profile. At least, some very small thickness must be input together with appropriate characteristics, so that the continuity of individual strata in the complete model of the subsoil is not interrupted.

The program itself then interpolates both the surface of the terrain ("the digital model of the terrain" can be displayed – see fig.8), the level of each geological stratum and all geomechanical characteristics.

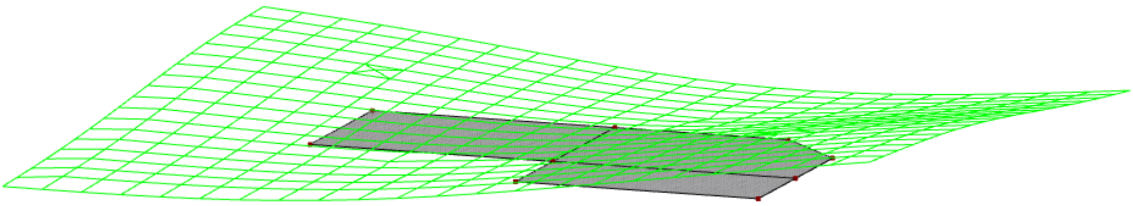


Fig.8 Display of the terrain

The interpolation is very robust and creates a smooth function surface from practically any input. It means that even a rough terrain can be easily modelled. In plan view, the surface is automatically defined by a rectangle whose edges are 10 m from the foundation structure. This makes it possible to avoid "sharp" transitions between geological zones that could – due to "step-changes" in stiffness – devaluate the determined supporting conditions in the given places and thus produce unreal internal forces in the superstructure.

As the user has defined the bore-holes and as the model of the superstructure has been already created, the program can in every place determine the corresponding depth of excavation, in which also the thickness of the foundation structure is taken into account. The same applies to the depth of excavation determined for the calculation of the C^S parameters, as well as to the correct determination of the surcharge loading level that models the contact stress for the given load combination.

The calculation of the settlement of the subsoil and subsequent determination of the C^S parameters is performed in a standard way using an iterative process. The result of this process is the state in which the contact stress or displacement u_z in two subsequent iterations does not change significantly. For that reason, the following quadratic norms are evaluated in every j-th iteration:

$$\varepsilon_\sigma = \frac{\sum_{i=1}^{n}\left(\sigma_{z,i,j} - \sigma_{z,i,j-1}\right)^2 A_i}{\sum_{i=1}^{n}\left|\sigma_{z,i,j} \cdot \sigma_{z,i,j-1}\right| A_i} \tag{25}$$

$$\varepsilon_u = \frac{\sum_{i=1}^{n} \left(u_{z,i,j} - u_{z,i,j-1}\right)^2 A_i}{\sum_{i=1}^{n} \left|u_{z,i,j} \cdot u_{z,i,j-1}\right| A_i} \tag{26}$$

where

n number of nodes

$\sigma_{z,i}$ contact stress in node i

A_i area corresponding to node i

$u_{z,i}$ global displacement of node i in the z-direction

The iterative calculation is stopped if $\varepsilon_\sigma < 0.01$ or $\varepsilon_u < 0.001$.

Under these conditions, the settlement is "proclaimed to be tuned" and further we deal only with the results for the superstructure. It means that we are interested in the deformation, internal forces and stress in the building.

If any problem occurs during the iterative calculation – e.g. the problem does not iterate, the number of iterations is too large, etc. – it is suitable to display the distribution of the contact stress in individual iterations, which may often help find the cause of the unfavourable behaviour. Moreover, we can display the C^S parameters calculated by the SOILIN module (see fig.9) and a table with the settlement of the subsoil. This table is useful e.g. to find out the settlement of points outside of the foundation structure.

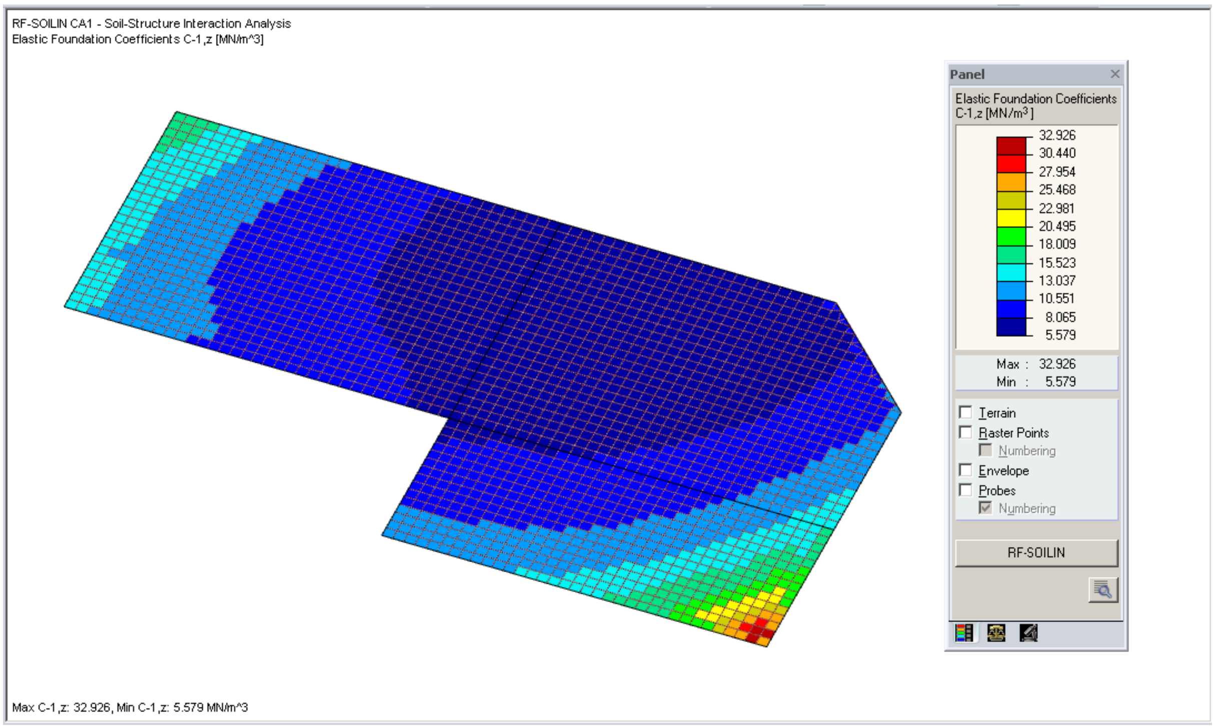


Fig.9 Calculated parameter C_{1z}

Conclusion

It is clear from the previous chapters that correct modelling of the structure-soil interaction is not a simple issue. Even the selection of appropriate input data for the subsoil is not problem-free. The physical model of the soil is strongly nonlinear and, therefore, the complete solution of the structure-soil interaction is significantly influenced by this factor. In addition, it is necessary to realise that the interaction parameters that model the supporting conditions are not constants (of the subsoil), but that they depend on the whole range of factors and, consequently, they reach different value in different place. And last, but not least, the model must take into account also the effect of the surrounding subsoil and neighbouring buildings.

RFEM with the integrated SOILIN module represents a versatile tool which can successfully handle all the above-mentioned pitfalls and produce reasonable results.

Acknowledgements

This research was conducted with the financial support of the Ministry of Education of the Czech Republic, project No. MSM0021630519. Authors acknowledge also the grant No. 103/08/0752 from the Grant Agency of Czech Republic.

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